AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022

SECONDARY 3 EXPRESS

Name:	Class:	Register No.:

MATHEMATICS **4052/01**
Paper 1 **28 September 2022**
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, gives the answer to 3 significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use
/90

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 A car travels at an average speed of 63.5 km/h for 2.45 hours.

(a) By rounding these numbers correct to 1 significant figure, find an estimate of the distance travelled by the car.
Show the numbers you use.

Answer ..... km [1]

(b) Without doing any further calculation, explain why the actual distance travelled by the car is greater than the answer to **part (a)**

Answer .....

..... [1]

---

2 The length of a rectangle is $2p$ cm and its breadth is p cm.
When the length of the rectangle is increased by 25% and breadth is decreased by 25%, calculate the percentage decrease in its area.

Answer ..... % [2]

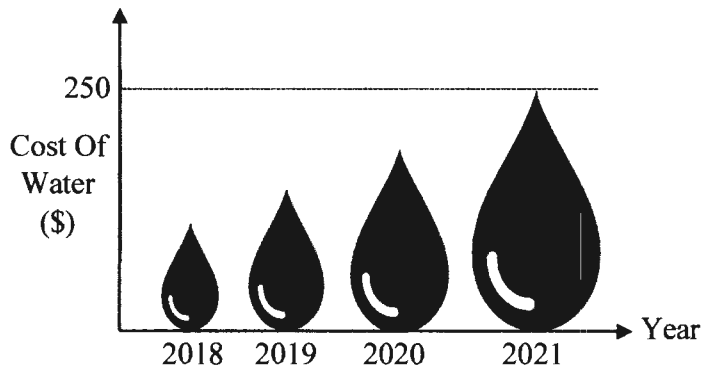
---

3 Simplify $\frac{3}{\sqrt[3]{8p^6}} \div \frac{p^5}{6}$.

Answer ..... [2]

---

4 Kenny draws this graph to show his annual water bill for each of the last four years.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

.....

.....

..... [2]

---

5 Given that $\left(\frac{32}{4^n}\right)^{-1} = \sqrt{64^n}$, find the value of n.

Answer $n =$ ..... [2]

---

6 Simplify $\frac{9x^2-4}{3x^2-10x-8}$.

Answer ..... [2]

---

7 **(a)** Factorise completely $6x^2y - 1 + 3x - 2xy$.

Answer ..... [2]

(b) Rearrange the formula $m = \dfrac{3 + p^2}{p^2 - q}$ to make p the subject.

Answer $p =$ ..... [3]

---

8 **(a)** Find the prime factors of 1176, giving your answer in index form.

Answer ..... [1]

(b) Two integers, A and B, can be written as product of prime factors.

$$A = 3 \times p \times q^{r+1} \qquad B = 3 \times p^3 \times q^r$$

The lowest common multiple (LCM) of A and B is 1176.

(i) Write down the value of p, q and r.

Answer $p =$ .....

$q =$ .....

$r =$ ..... [3]

(ii) Find the highest common factor (HCF) of A and B.

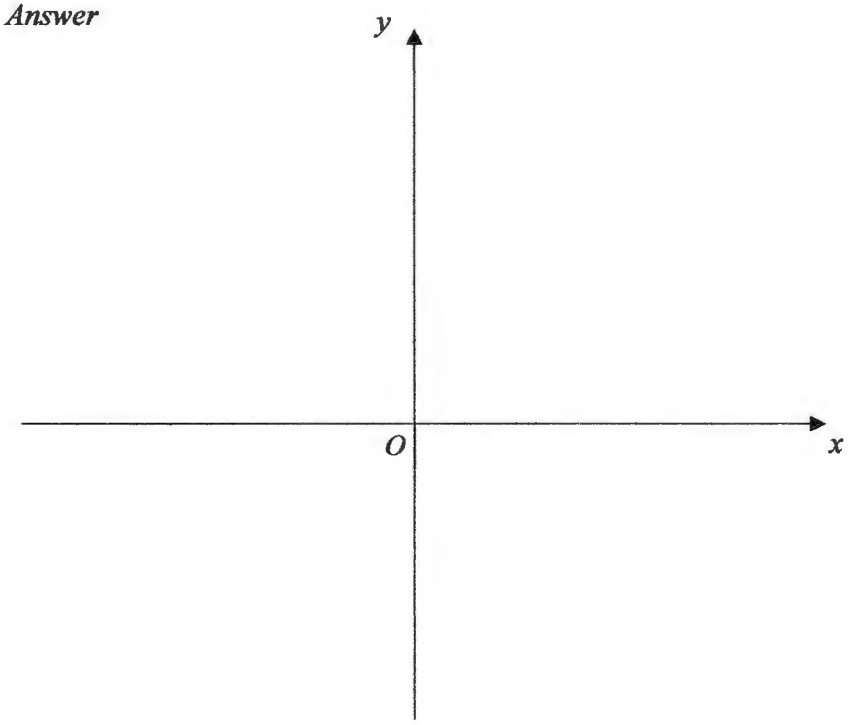
Answer ..... [1]

9 **(a)** Express $5+4x-x^2$ in the form $-(x+a)^2+b$.

Answer ..... [2]

(b) Hence sketch the graph of $y=5+4x-x^2$, indicating clearly the coordinates of the points where the graph crosses the axes and the turning point.

Answer



[2]

(c) Explain why the equation $7+4x-x^2=k$ does **not** have solutions for some values of k.

Answer .....

.....

.....

..... [2]

10 Andrew invested \$15 000 at a rate of r % per year compound interest.
At the end of 3 years, the money had earned total interest of \$1872.96.
Find the value of r.

Answer $r =$ ..... [3]

---

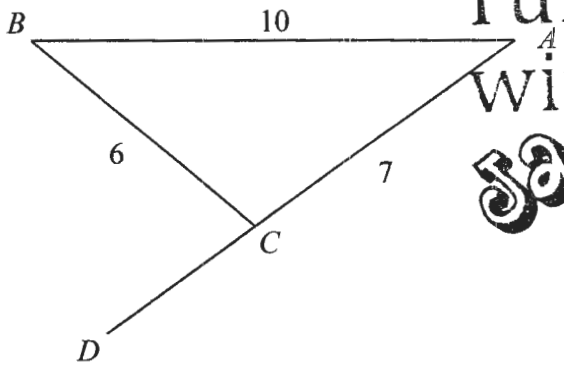
11 **(a)** List all the prime numbers that satisfy $-2 \leq x < 5$.

Answer ..... [1]

(b) Solve the inequalities $-9 < 2x - 3 \leq 7$.

Answer ..... [2]

12



DCA is a straight line.
$AB = 10$ cm, $BC = 6$ cm and $AC = 7$ cm.

(a) Find $\cos \angle BCA$, giving your answer as a fraction.

Answer ..... [2]

(b) Find $\cos \angle BCD$, giving your answer as a fraction.

Answer ..... [1]

13 **(a)** Construct triangle ABC such that $AC = 6.8$ cm and $BC = 9$ cm.
The point T is such that it is equidistant from A and B.
The point T is also equidistant from AB and BC.
Line AB has been drawn for you.

Find and label T.

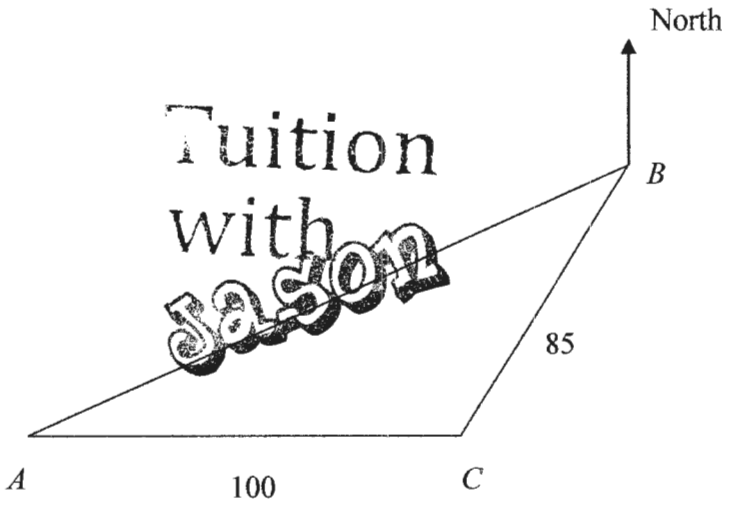
Answer

[3]

(b) Shade the region inside ABC that is closer to AB than to BC and closer to A than to B.

[1]

14



The diagram shows the positions of 3 towns, A, B and C.
The bearing of A from B is 245°.
C is due east of A.
B is 85 km from C and A is 100 km from C.

(a) Calculate angle BAC.

Answer Angle BAC = ... [2]

(b) Find the bearing of C from B.

Answer ..... [2]

15 A bag contains some red balls, green balls and yellow balls.
The probability of choosing a red ball at random is 0.4.
The probability of choosing a green ball at random is 0.15.

(a) Find the probability of choosing a yellow ball.

Answer .....[1]

(b) In the bag there are 5 more yellow balls than red balls.
Find the total number of balls in the bag.

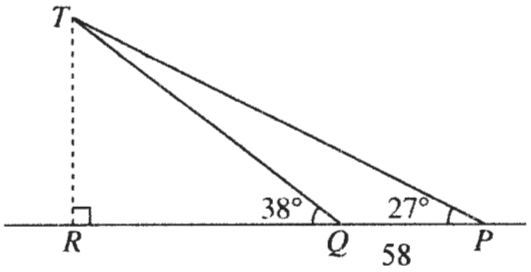
Answer ..... [2]

---

16 The force of attraction, F newtons, between the magnets is inversely proportional to the square of the distance, d cm, between the magnets.
The distance is reduced by half its original value.
Calculate the percentage increase in the force of attraction between the magnets.

Answer .....% [3]

17

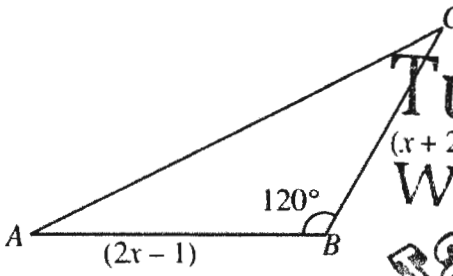


The diagram shows a vertical lighthouse TR.
The angle of elevation of T from P is 27°.
The angle of elevation of T from Q is 38°.
The distance between P and Q is 58 m.

Find the height of the lighthouse TR.

Answer ..... m [3]

18



In triangle ABC, $AB = (2x-1)$ cm, $BC = (x+2)$ cm and angle $ABC = 120°$.

(a) Given that $AC = \sqrt{127}$ cm, form an equation in x and show that it reduces to $7x^2 + 3x - 124 = 0$.

Answer

[3]

(b) Solve the equation $7x^2 + 3x - 124 = 0$.

Answer $x =$ ..... or $x =$ .....[3]

(c) Calculate the area of triangle ABC.

Answer ..... cm^2 [2]

---

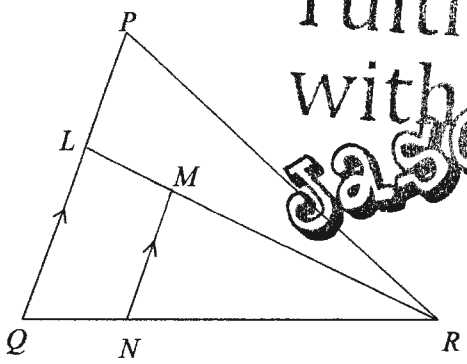
19 **(a)** In this question, use the fact that 1 light year = 9.46×10^{15} metres.
The distance of the star Sirius from the Sun is 8.6 light years.
Calculate the distance, in kilometres, of Sirius from the Sun.
Give your answer in standard form.

Answer .....km [2]

(b) The distance of the star Proxima Centauri from the Sun is 3.97×10^{13} km.
A space probe travels at 50 000 km/h.
Calculate the time taken for the probe to travel from the Sun to Proxima Centauri.
Give your answer in years, correct to three significant figures.

Answer .....years [2]

20



PQ and MN are parallel lines.
N lies on the line QR such that $QN : NR = 1 : 4$.

(a) Show that triangles QLR and NMR are similar.
Give a reason for each statement you make.

Answer .....

.....

.....

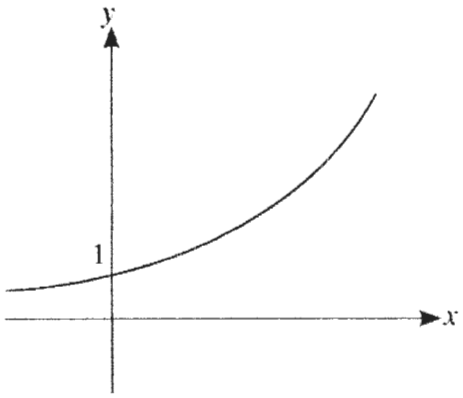
.....

..... [2]

(b) The area of triangle QLR is 50 cm^2.
Calculate the area of $QLMN$.

Answer ..... cm^2 [3]

21 **(a)**



Write down a possible equation for the graph above.

Answer .....[1]

(b) A line $y = k$ is drawn on the same axes above.
The line does not intersect with the graph.

Write down the range of values of k.

Answer .....[1]

22 The equation of line l is $3x - y = 5$.
P is the point (5, 12).

(a) Determine if point P lie on the line l.

Answer

[1]

(b) Find the equation of the line passing through P that does not intersect line l.

Answer .....[2]

(c) Q is a point which lies on the line l.
The length of the line segment PQ is $\sqrt{130}$ units.

Find the two possible coordinates of Q.

Answer (..... ,.....) or (..... , ...) [4]

23 The table shows the heights of 32 students in 3E4.

Height, h (in cm)	Frequency
$140 \le h < 150$	$x-3$
$150 \le h < 160$	$2x+1$
$160 \le h < 170$	$3x+1$
$170 \le h < 180$	$x-2$

(a) Find the value of x.

Answer $x =$ ..... [2]

(b) Hence, calculate an estimate of the mean height of the students.

Answer .....cm [1]

24 Five numbers are given in a list.

6 10 27 2 10

(a) An extra number x is added to the list such that the mean increases by 3.
Find the value of x.

Answer $x =$ ..... [1]

(b) An extra number y is added to the original list such that the median decreased by 2.
Find the value of y.

Answer $y =$ ..... [1]

(c) A number is removed from the original list so that the median and range do not change.
Write down this number

Answer ..... [1]

25 The table shows the times (in minutes) taken by a class of students to travel from home to school daily.

Stem	Leaf					
0	5	8	9			
1	2	2	2	7	7	9
2	0	3	6	8		
3	0	2	5	7	8	
4	1	3	5	6		
5	4	6				

Key

1 | 2 represents 12 minutes

(a) Find the median time.

Answer .....minutes [1]

(b) Find the percentage of students who took more than 30 minutes to travel to school daily.

Answer ..... % [1]

(c) It was discovered that the times had been recorded incorrectly.
The correct times were all 2 minutes less than those recorded.
Explain how the median and range of the times have been affected by this error.

Answer .....

.....

.....

.....

.....[2]

End of Paper

Setter: Mr Teo Lip Seng

AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022

SECONDARY 3 EXPRESS

Name:	Class:	Register No.:

MATHEMATICS **4052/02**
Paper 2 **29 September 2022**
Candidates answer on the Question Paper. **2 hour 15 minutes**

READ THESE INSTRUCTIONS FIRST

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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use
/ 90

This document consists of **21** printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Alpha Data Company has the following promotion for tourists.

	7-day tourist SIM card	12-day tourist SIM card
First 100 GB of data	$\$ p$	$\$(3p-4)$
Every additional GB of data	$\$ (q+5)$	$\$ q$

Tom bought a 7-day tourist SIM card and used 105 GB of data.
$62 was deducted from his credit card bill for his data usage after his vacation.
Claire bought a 12-day tourist SIM card and used 112 GB of data.
$92 was deducted from her credit card bill for her data usage after her vacation.

(a) Write down two simultaneous equations in terms of p and q, to represent this information.

Answer ..... [2]

(b) Solve the simultaneous equations.

Answer $p =$ .....

$q =$ ..... [3]

2 **(a)** Write as a single fraction in its simplest form

(i) $\dfrac{2}{(x-5)^2} - \dfrac{3}{(5-x)}$,

Answer ..... [2]

(ii) $\dfrac{a^2-ab^2}{ab} \times \dfrac{ab^2}{ab-a^2}$.

Answer ..... [2]

(b) Explain why $81x^2+4=0$ has no real solutions.

.....

.....

..... [1]

(c) Solve $\frac{3x-1}{2}-8=\frac{x+3}{5}$.

Answer $x =$ ..... [3]

3 **(a)** Simplify $6^{2n+1} \times 8^n \times 3^{2n}$.

Answer ..... [3]

(b) Solve $4^x\left(5^{2x}\right)=10$.

Answer $x =$ ..... [3]

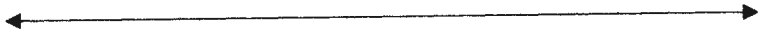
(c) The reciprocal of $\sqrt[3]{32}$ is 4^n.
Find the exact value of n.

Answer $n =$ ..... [2]

(d) List all the integers that satisfy the inequalities

$3 \leq x-3 \leq 7$ and $\frac{x-1}{3} < \frac{2x+1}{5} < 4$.

Represent your answer on a number line.



Answer ..... [5]

4 Rui has a map drawn to the scale 1 : 200 000.

(a) The distance on the map between Gardens by the Bay and Night Safari is 14.1 cm
Calculate the actual distance, in kilometres, between Gardens by the Bay and Night Safari.

Answer ..... km [2]

(b) Gardens by the Bay covers an area of 1.01 km^2.
Find the area, in square centimetres, covered by Gardens by the Bay on the map.

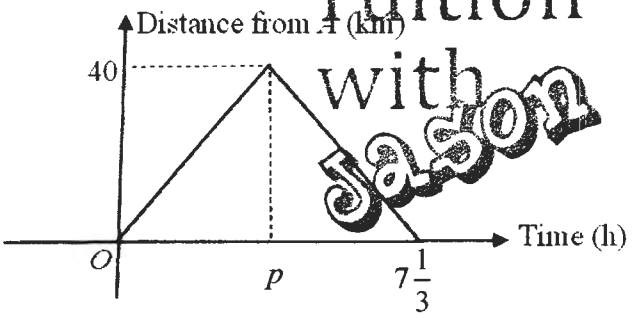
Answer ..... cm^2 [2]

(c) The area covered by Gardens by the Bay on Ting's map is 4 times that on Rui's map.
Find the scale of Ting's map, giving your answer in the form 1 : *n*.

Answer 1 : ..... [3]

[Turn over for Question 5]

5



A cyclist cycled a distance of 40 km from A to B at an average speed of x km/h for p hours where x and p are constants.
He then cycled from B to A by the same route but at an average speed of 2 km/h slower.
The total time taken by the cyclist for the whole journey was $7\frac{1}{3}$ hours.

(a) Express p in terms of x.

Answer $p =$ ..... [1]

(b) Show that $\frac{22}{3} - p = \frac{40}{x-2}$ when he cycled from B to A.

Answer

[2]

(c) Using the results in parts **(a)** and **(b)**, show that $11x^2 - 142x + 120 = 0$.

Answer

[3]

(d) Solve $11x^2 - 142x + 120 = 0$.

Answer $x =$ ..... or $x =$ ..... [3]

(e) Explain why one of the solutions in part **(d)** must be rejected.

Answer

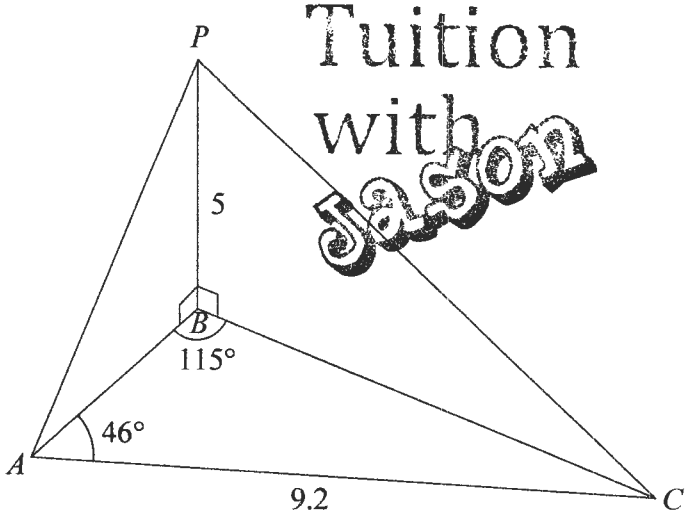
[1]

(f) Hence find the possible value of x and p.

Answer $x =$ .....

$p =$ ..... [2]

6



The diagram shows a field ABC on horizontal ground.
BP is a vertical pole vertically above B.
$AC = 9.2$ m, $BP = 5$ m, angle $BAC = 46°$ and angle $ABC = 115°$.

(a) Calculate AB.

Answer ..... m [3]

(b) Tom walks along AC.
Find Tom's distance from A where he is equidistant from A and B.

Answer ..... m [3]

(c) The angle of depression of C from P is 34.4°.
Calculate CP.

Answer ..... m [2]

(d) Calculate angle ACP.

Answer Angle ACP = ... [3]

7 A group of researchers brought 180 kangaroos to an island and tracked the number of kangaroos over a few years.
Some corresponding values of x and y are given in the table below.

x (Number of years)	0	0.5	1	2	4	5	6	6.5	7	7.5
y (Number of kangaroos)	180	177	175	172	172	175	180	183	187	191

(a) On the grid opposite, plot the points given in the table and join them with a smooth curve. [3]

(b) Use your graph to estimate the number of kangaroos at the third year, correct your answer to the nearest integer.

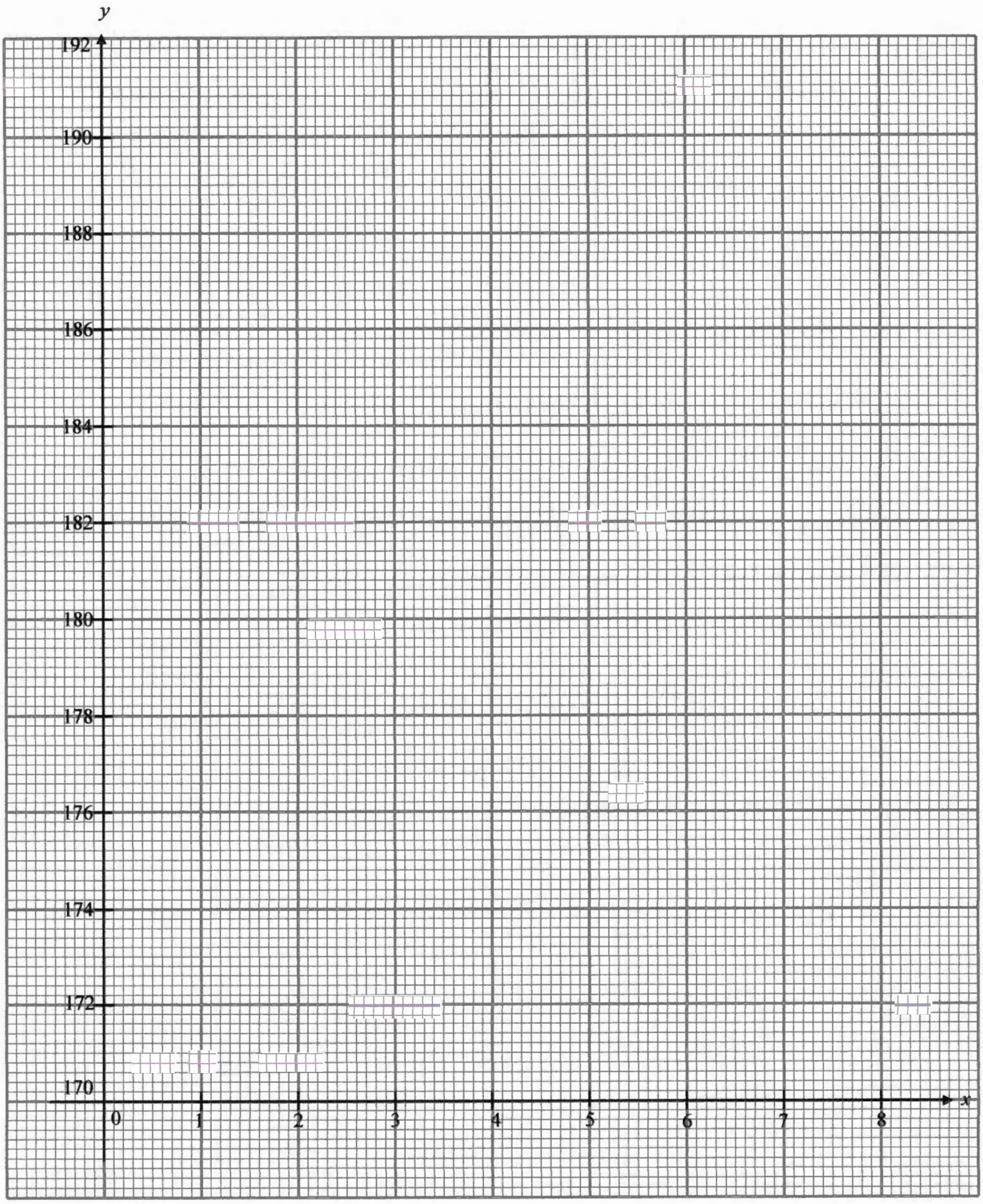
Answer ..... [1]

(c) By drawing a tangent, find the gradient of the curve at $x = 2$.

Answer ..... [2]

(d) The researchers found out that the number of kangaroos can be modelled after a quadratic function of the form $y = (x + a)^2 + b$.
Find the value of a and b.

Answer $a =$ ....., $b =$ ..... [2]


y
192
190
188
186
184
182
180
178
176
174
172
170
0
1
2
3
4
5
6
7
8
x

(e) The researchers then brought 180 kangaroos of a different breed to another island and observed that the number of kangaroos increased at a constant rate of 1 per year.
Write down an equation connecting the number of kangaroos, y, and the number of years, x.

Answer $y =$ ..... [1]

(f) On the same grid, draw the graph of the equation in part **(e)** for $4 \leq x \leq 8$. [1]

(g) Explain the significance of the point of intersection of the two graphs.

Answer .....

..... [1]

8 Part of a pattern of numbers is shown in the table below.

	C_1	C_2	C_3	C_4	C_5	C_6
Row 1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13
Row 3	14	15	16	17	18	19
Row 4	20	21	22	23	24	25
Row 5	26	27	28	29	30	31
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Row n		a	b			
Row $n+1$		r	s			

(a) Express b, r and s in terms of a.

Answer $b =$ .....

$r =$ .....

$s =$ ..... [3]

(b) Show that the difference between rb and as is always 6.

Answer

[2]

(c) **(i)** Express a in terms of n.

Answer $a =$ ..... [1]

(ii) Express b in terms of n.

Answer $b =$ ..... [1]

(d) **(i)** Explain why *ab* will be in column number C_5, for any value of *n*.

Answer .....

..... [2]

(ii) Which column would *rs* lies in?

Answer ..... [1]

9

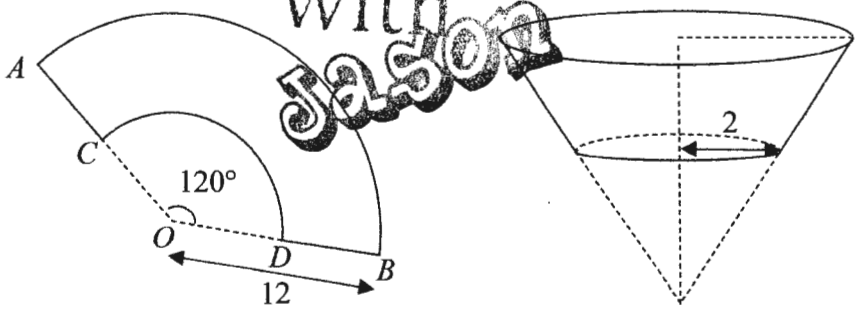


Diagram I Diagram II

In diagram I, OAB is a sector of a circle, centre O, with radius 12 cm.
The region $CABD$ is cut from the sector and folded to form the frustum in Diagram II.
The small cone in diagram II has a radius of 2 cm.

(a) (i) Find the radius of the top of the frustum.

Answer ..... cm [2]

(ii) Show that the volume of the frustum is 165.87 cm^3 when rounded to 5 significant figures.

Answer

[4]

(b)

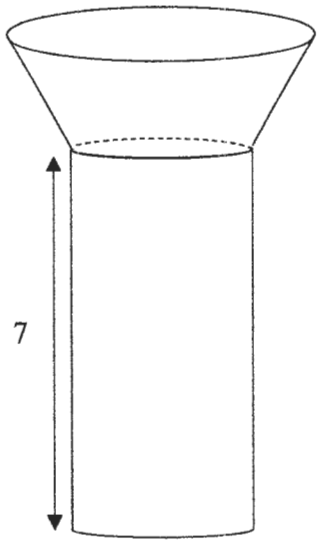
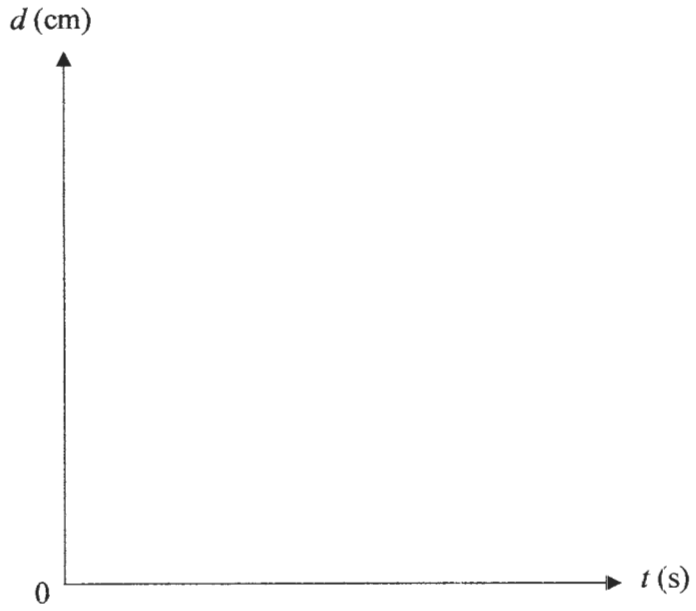


Diagram III

A cylinder is attached to the frustum to form a vase as shown in Diagram III.
Water is poured into the vase at a constant rate.

(i) On the axes, sketch the graph of the depth, d cm, of the water against time, t seconds.

Answer



[2]

(ii) The manufacturer wants to modify the vase in diagram III such that the height of the cylinder is $\frac{6}{5}$ of its current height.

It takes 10 seconds to fill up the vase in diagram III.
The manufacturer claims that the time taken to fill the modified vase will approximately be 2.5 seconds more.
Assuming that the water is being filled up at the same rate, is the manufacturer correct?
Justify your answers with calculations.

Answer

[5]

End of Paper

AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022

SECONDARY 3 EXPRESS

Name: Mark Scheme	Class:	Register No.:

MATHEMATICS **4052/01**
Paper 1 **28 September 2022**
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, gives the answer to 3 significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use
/90

This document consists of 23 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 A car travels at an average speed of 63.5 km/h for 2.45 hours.

(a) By rounding these numbers correct to 1 significant figure, find an estimate of the distance travelled by the car.
Show the numbers you use.

Distance $= S \times T$
$= 60 \times 2$
$= 120$ km [B1]

Answer ..... km [1]

(b) Without doing any further calculation, explain why the actual distance travelled by the car is greater than the answer to **part (a)**

Answer B

Both the speed and time were rounded down, therefore the actual distance is greater than the answer in part (a) [B1]

[1]

---

2 The length of a rectangle is $2p$ cm and its breadth is p cm.
When the length of the rectangle is increased by 25% and breadth is decreased by 25%, calculate the percentage decrease in its area.

Original area $= 2p^2$
New area $= 1.25\,(2p) \times 0.75p$
$= 1.875p^2$ [M1]

Percentage decrease $= \dfrac{2-1.875}{2} \times 100$
$= 6.25\%$ [A1]

Answer ..... % [2]

---

3 Simplify $\frac{3}{\sqrt[3]{8p^6}} \div \frac{p^5}{6}$.

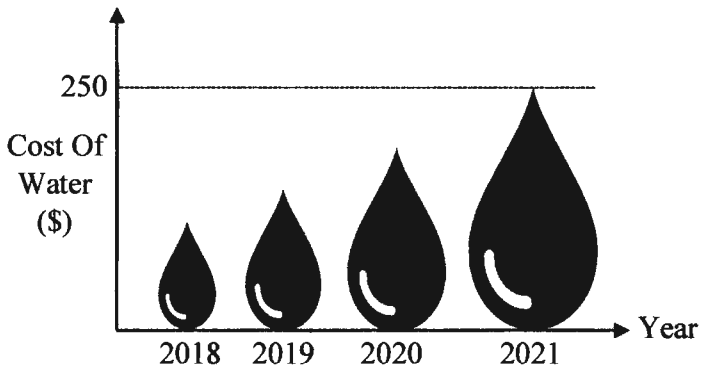
$= \frac{3}{2p^2} \times \frac{6}{p^5}$ [M1]

$= \frac{9}{p^7}$ [A1]

Answer ..... [2]

---

4 Kenny draws this graph to show his annual water bill for each of the last four years.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

There is no defined scale on the vertical axis. [B1]
The cost of water in year 2021 maybe misinterpreted as 4 times the cost of water in year 2018 [B1]

[2]

---

5 Given that $\left(\frac{32}{4^n}\right)^{-1} = \sqrt{64^n}$, find the value of n.

$\frac{4^n}{32} = 8^n$

$\frac{2^{2n}}{2^5} = 2^{3n}$ [M1 - same base]

$2^n = 2^{-5}$

$n = -5$ [A1]

Answer $n =$ ..... [2]

---

6 Simplify $\frac{9x^2 - 4}{3x^2 - 10x - 8}$.

$= \frac{(3x)^2 - 2^2}{(3x+2)(x-4)}$

$= \frac{(3x+2)(3x-2)}{(3x+2)(x-4)}$ [M1- difference of square]

$= \frac{3x-2}{x-4}$ [A1]

Answer ..... [2]

---

7 **(a)** Factorise completely $6x^2y-1+3x-2xy$.

$6x^2y-1+3x-2xy$
$=3x(2xy+1)-(2xy+1)$ [M1]
$=(3x-1)(2xy+1)$ [A1]

Answer ..... [2]

(b) Rearrange the formula $m=\dfrac{3+p^2}{p^2-q}$ to make p the subject.

$m(p^2-q)=3+p^2$
$mp^2-mq=3+p^2$ [M1]
$mp^2-p^2=3+mq$
$p^2(m-1)=3+mq$ [M1]
$p=\pm\sqrt{\dfrac{3+mq}{m-1}}$ [A1]

Answer $p=$ ..... [3]

---

8 **(a)** Find the prime factors of 1176, giving your answer in index form.

$$1176 = 2^3 \times 3 \times 7^2 \quad \text{[B1]}$$

Answer ..... [1]

(b) Two integers, A and B, can be written as product of prime factors.

$$A = 3 \times p \times q^{r+1} \qquad B = 3 \times p^3 \times q^r$$

The lowest common multiple (LCM) of A and B is 1176.

(i) Write down the value of p, q and r.

Answer p = ...2...[B1]

q = ...7...[B1]

r = ...1...[B1] [3]

(ii) Find the highest common factor (HCF) of A and B.

$$\begin{aligned} \text{HCF} &= 2 \times 3 \times 7 \\ &= 42 \end{aligned} \quad \text{[B1]}$$

Answer ..... [1]

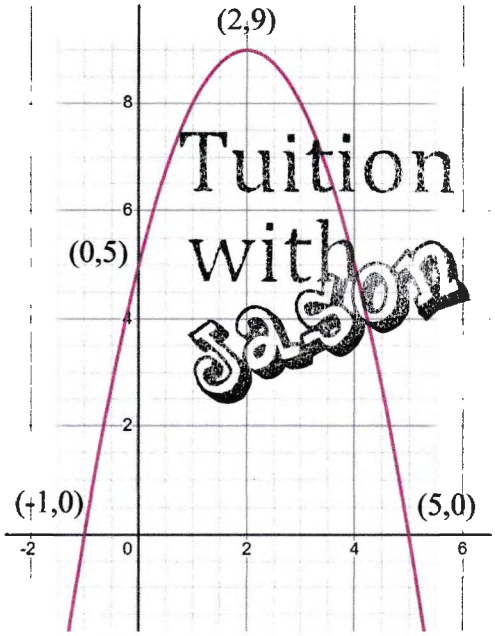
9 **(a)** Express $5+4x-x^2$ in the form $-(x+a)^2+b$.

$=-(x^2-4x-5)$
$=-[(x-2)^2-2^2-5]$ [M1]
$=-(x-2)^2+9$ [A1]

Answer ..... [2]

(b) Hence sketch the graph of $y=5+4x-x^2$, indicating clearly the coordinates of the points where the graph crosses the axes and the turning point.

Answer



P1 – Shape
P1 – x-axis intercepts and turning point

[2]

(c) Explain why the equation $7+4x-x^2=k$ does **not** have solutions for some values of k.

Answer
$7+4x-x^2=k$
$5+4x-x^2=k-2$ [B1]
Solution of $7+4x-x^2=k$ can be found from the intersection of $y=5+4x-x^2$ and $y=k-2$
From the graph of $y=5+4x-x^2$, any horizontal line above $y=9$ will not have any intersection.
Hence there will not be any solutions for $k-2>9$ ie $k>11$ [B1]
..... [2]

10 Andrew invested \$15 000 at a rate of r % per year compound interest. At the end of 3 years, the money had earned total interest of \$1872.96. Find the value of r.

$15000+1872.96=15000(1+\frac{r}{100})^3$ [M1]

$\frac{16872.96}{15000}=(1+\frac{r}{100})^3$

$(1+\frac{r}{100})=1.04$ [M1]

$r=4$ [A1]

Answer $r =$ ..... [3]

---

11 **(a)** List all the prime numbers that satisfy $-2 \le x < 5$.

Answer .....2 , 3 [B1]..... [1]

(b) Solve the inequalities $-9 < 2x-3 \le 7$.

$-9 < 2x-3$

$2x > -6$

$x > -3$ [M1]

$2x-3 \le 7$

$2x \le 7+3$

$2x \le 10$

$x \le 5$

$-3 < x \le 5$ [A1]

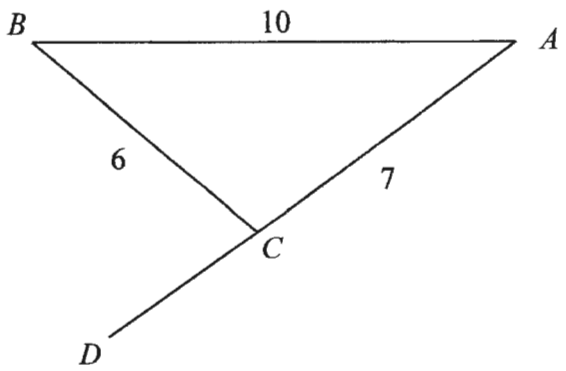
OR

$-6 < 2x \le 10$

$-3 < x \le 5$

Answer ..... [2]

12



DCA is a straight line.
$AB = 10$ cm, $BC = 6$ cm and $AC = 7$ cm.

(a) Find $\cos \angle BCA$, giving your answer as a fraction.

$10^2 = 6^2 + 7^2 - 2(6)(7)\cos \angle BCA$ [M1]

$\cos \angle BCA = -\frac{15}{84}$

$= -\frac{5}{28}$ [A1]

Answer ..... [2]

(b) Find $\cos \angle BCD$, giving your answer as a fraction.

$\cos \angle BCD = \frac{5}{28}$ [B1]

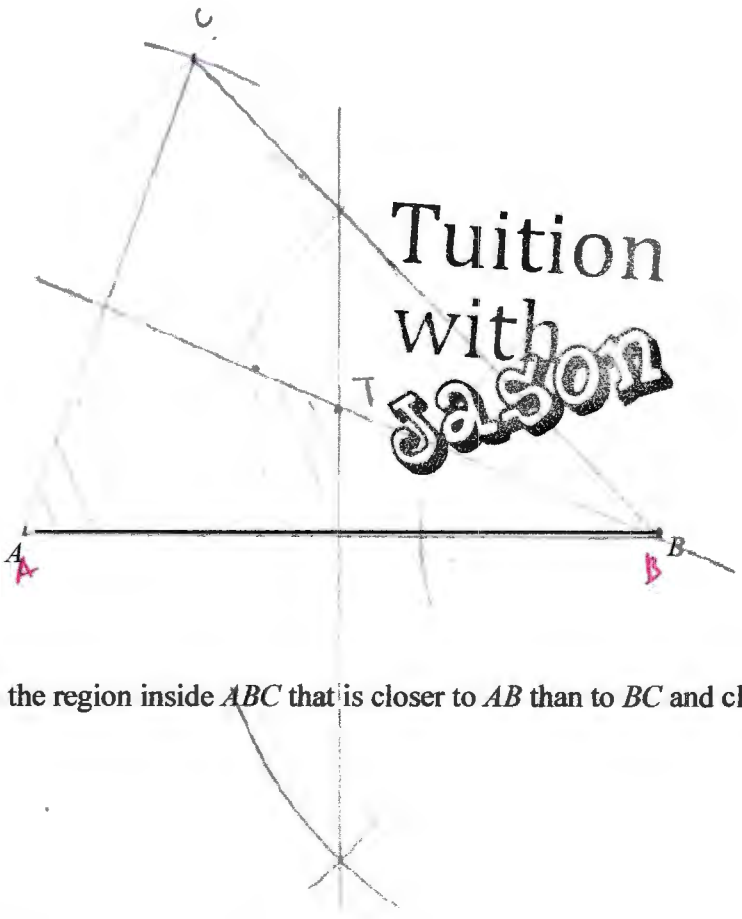
Answer ..... [1]

13 **(a)** Construct triangle ABC such that $AC = 6.8$ cm and $BC = 9$ cm.
The point T is such that it is equidistant from A and B.
The point T is also equidistant from AB and BC.
Line AB has been drawn for you.

Find and label T.

Answer

B1 – Construction of line bisector
B1 – Construction of angle bisector
B1 – locate T

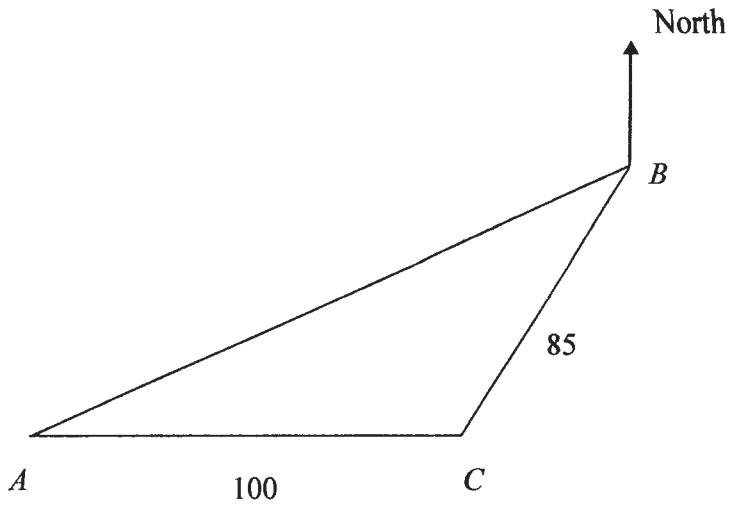


[3]

(b) Shade the region inside ABC that is closer to AB than to BC and closer to A than to B.

[1]

14



The diagram shows the positions of 3 towns, A, B and C.
The bearing of A from B is 245°.
C is due east of A.
B is 85 km from C and A is 100 km from C.

(a) Calculate angle BAC.

$\angle TBA = 360 - 245$ ($\measuredangle$s at a point)
$= 115°$
$\angle UAB = 180 - 115$ (Int $\measuredangle$s, UA//TB)
$= 65°$ [M1]
$\angle BAC = 90 - 65$
$= 25°$ [A1]

$\angle a = 270° - 245°$
or $= 25°$ [M1]
$\angle BAC = \angle a$(alt $\angle$s, // lines) [A1]

$\angle ABX = 245° - 180°$
$= 65°$ [M1]
or $\angle BAC = \angle BAX$
$= 180° - 90° - 65°$
$= 25°$($\angle$ sum of triangle) [A1]

Answer ..... [2]

(b) Find the bearing of C from B.

$$\frac{100}{\sin \angle ABC} = \frac{85}{\sin \angle BAC}$$

$\sin \angle ABC = 0.4972$

$\angle ABC = 29.8°$ [M1]

Bearing of C from B $= 245 - 29.8$

$= 215.2°$ [A1]

** deduct 1 mark for the entire question for incomplete reason **

Answer ..... [2]

---

15 A bag contains some red balls, green balls and yellow balls.
The probability of choosing a red ball at random is 0.4.
The probability of choosing a green ball at random is 0.15.

(a) Find the probability of choosing a yellow ball.

P(yellow) $= 1 - 0.4 - 0.15$

$= 0.45$ [B1]

45% is not accepted

Answer .....[1]

(b) In the bag there are 5 more yellow balls than red balls.
Find the total number of balls in the bag.

let total number number of balls be x

$0.45x - 0.4x = 5$ [M1]

$x = 100$ [A1]

Answer .....[2]

16 The force of attraction, F newtons, between the magnets is inversely proportional to the square of the distance, d cm, between the magnets.
The distance is reduced by half its original value.
Calculate the percentage increase in the force of attraction between the magnets.

$F = \frac{k}{d^2}$

new $d = 0.5d$

$F_{new} = \frac{k}{(0.5d)^2}$

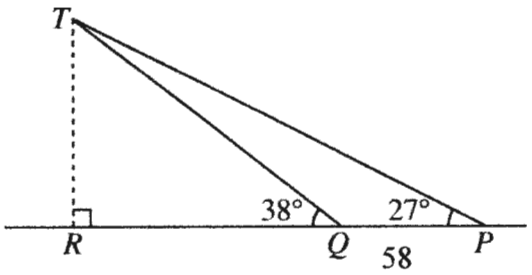
$= 4(\frac{k}{d^2})$ [M1]

Percentage increase $= \frac{4-1}{1} \times 100$ [M1]

$= 300\%$ [A1]

Answer .....% [3]

17



The diagram shows a vertical lighthouse TR.
The angle of elevation of T from P is 27°.
The angle of elevation of T from Q is 38°.
The distance between P and Q is 58 m.

Find the height of the lighthouse TR.

$$\tan 27 = \frac{TR}{RQ+58}$$

$$\tan 38 = \frac{TR}{RQ} \qquad \text{[M1]}$$

$$TR = RQ(\tan 38)$$

$$\tan 27 = \frac{RQ(\tan 38)}{RQ+58}$$

$$(RQ+58)\tan 27 = RQ(\tan 38)$$

$$RQ = \frac{58(\tan 27)}{\tan 38 - \tan 27} \qquad \text{[M1]}$$

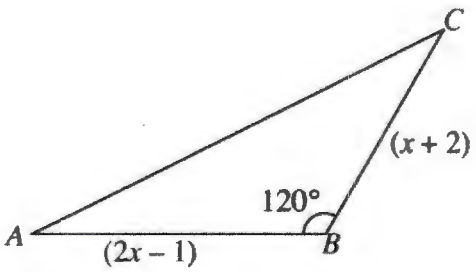
$$RQ = 108.7446$$

$$TR = RQ(\tan 38)$$

$$TR = 85.0m \qquad \text{[A1]}$$

Answer ..... m [3]

18



In triangle ABC, $AB = (2x-1)$ cm, $BC = (x+2)$ cm and angle $ABC = 120°$.

(a) Given that $AC = \sqrt{127}$ cm, form an equation in x and show that it reduces to $7x^2 + 3x - 124 = 0$.

Answer

$$AC^2 = (2x-1)^2 + (x+2)^2 - 2(2x-1)(x+2)\cos 120° \qquad \text{[M1]}$$

$$127 = 4x^2 - 4x + 1 + x^2 + 4x + 4 - 2(2x^2 + 3x - 2)(-0.5) \qquad \text{[M1]}$$

$$127 = 5x^2 + 5 + 2x^2 + 3x - 2$$

$$7x^2 + 3x - 124 = 0 \quad \text{(shown)} \qquad \text{[A1]}$$

[3]

(b) Solve the equation $7x^2 + 3x - 124 = 0$.

$7x^2 + 3x - 124 = 0$

$(7x + 31)(x - 4) = 0$ [M1] or $x = \dfrac{-3 \pm \sqrt{3^2 - 4(7)(-124)}}{2(7)}$ [M1]

$x = -4\dfrac{3}{7}$ or $x = 4$ [A2] $x = -4\dfrac{3}{7}$ or $x = 4$ [A2]

$= -4.43$ (3 s.f)

Answer $x =$ ..... or $x =$ .....[3]

(c) Calculate the area of triangle ABC.

$Area = \dfrac{1}{2}(7)(6)\sin 120°$ [M1]

$= 18.1865$

$= 18.2$ [A1]

Answer ..... cm^2 [2]

---

19 **(a)** In this question, use the fact that 1 light year = 9.46×10^{15} metres.
The distance of the star Sirius from the Sun is 8.6 light years.
Calculate the distance, in kilometres, of Sirius from the Sun.
Give your answer in standard form.

8.6 light years $= 9.46 \times 10^{15} \times 8.6 \times 10^{-3}$ [M1]

$= 81.356 \times 10^{12}$

$= 8.1356 \times 10^{13}$ *km* [A1 or B2]

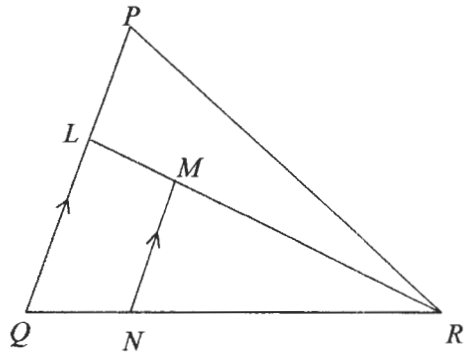
Answer .....km [2]

(b) The distance of the star Proxima Centauri from the Sun is 3.97×10^{13} km.
A space probe travels at 50 000 km/h.
Calculate the time taken for the probe to travel from the Sun to Proxima Centauri.
Give your answer in years, correct to three significant figures.

$$\begin{aligned} Time &= \frac{3.97\times10^{13}}{50000} \\ &= 7.94\times10^{8} \text{ hr} && [M1] \\ &= \frac{7.94\times10^{8} \text{ hr}}{24\times365} \\ &= 90639.269 \\ &= 90600 \text{ years} && [A1] \end{aligned}$$

Answer .....years [2]

20



PQ and MN are parallel lines.
N lies on the line QR such that $QN : NR = 1 : 4$.

(a) Show that triangles QLR and NMR are similar.
Give a reason for each statement you make.

Answer

$\angle LQR = \angle MNR$ (Corr. $\angle$s, $PQ // MN$)
$\angle LRQ = \angle MRN$ (Common angle) [B1]
Since 2 pairs of corresponding angles are equal [B1]
Triangles QLR and NMR are similar

[2]

(b) The area of triangle QLR is 50 cm^2.
calculate the area of $QLMN$.

$$\frac{\text{Area of triangle } MNR}{\text{Area of triangle } LQR} = (\frac{4}{5})^2 \qquad \text{[M1]}$$

$$\frac{\text{Area of triangle } MNR}{50} = \frac{16}{25}$$

$$\text{Area of triangle } MNR = 32 \text{ cm}^2 \qquad \text{[M1]}$$

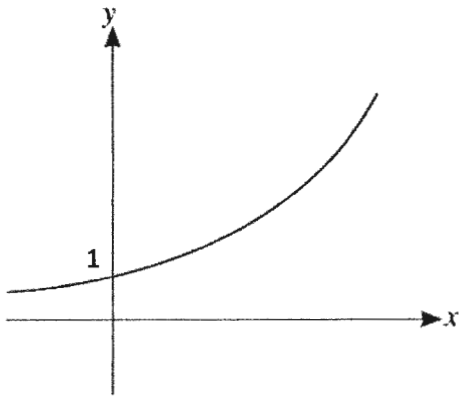
$$\text{Area of } QLMN = 50 - 32$$

$$= 18 \text{ cm}^2 \qquad \text{[A1]}$$

Answer ..... cm^2 [3]

---

21 **(a)**



Write down a possible equation for the graph above.

$y = 3^x$ [B1 accept suitable equation]

Answer .....[1]

(b) A line $y = k$ is drawn on the same axes above.
The line does not intersect with the graph.

Write down the range of values of k.

$k \leq 0$ [B1]

Answer .....[1]

---

22 The equation of line l is $3x - y = 5$.
P is the point (5, 12).

(a) Determine if point P lie on the line l.

$3x - y = 5$
$y = 3x - 5$
when $x = 5$
$y = 3(5) - 5$
$y = 10 \neq 12$
therefore P does not lie on the line l. [B1]

[1]

(b) Find the equation of the line passing through P that does not intersect line l.
Let this line be $y = mx + c$.
For this line not to intersect l, they must have the same gradient
therefore $m = 3$ [M1]
Equation of line : $y = 3x + c$
at P, $x = 5, y = 12$
$12 = 3(5) + c$
$c = -3$
$y = 3x - 3$ [A1]

Answer .....[2]

(c) Q is a point which lies on the line l.
The length of the line segment PQ is $\sqrt{130}$ units.
Find the two possible coordinates of Q.

Let coordinates of Q be $(x, 3x-5)$

$130 = (3x-5-12)^2 + (x-5)^2$ [M1]

$130 = (3x-17)^2 + (x-5)^2$

$130 = 9x^2 - 102x + 289 + x^2 - 10x + 25$

$5x^2 - 56x + 92 = 0$ [M1]

$(5x-46)(x-2) = 0$

$x = 2$ or $x = 9\frac{1}{5}$

$y = 1$ or $y = 22\frac{3}{5}$

The possible coordinates of Q are

$(2,1), (9\frac{1}{5}, 22\frac{3}{5})$ [A2]

Answer (..... ,.....) or (..... , ...) [4]

---

23 The table shows the heights of 32 students in 3E4.

Height, h (in cm)	**Frequency**
$140 \le h < 150$	$x-3$
$150 \le h < 160$	$2x+1$
$160 \le h < 170$	$3x+1$
$170 \le h < 180$	$x-2$

(a) Find the value of x.

$x-3+2x+1+3x+1+x-2 = 32$ [M1]

$7x-3 = 32$

$x = 5$ [A1]

Answer $x =$ ..... [2]

(b) Hence, calculate an estimate of the mean height of the students.

$$mean = \frac{(145\times 2)+(155\times 11)+(165\times 16)+(175\times 3)}{32}$$

$= 161.25$ [B1]

Answer .....cm [1]

24 Five numbers are given in a list.

6 10 27 2 10

(a) An extra number x is added to the list such that the mean increases by 3.
Find the value of x.

$\frac{x+55}{6}=14$

$x=29$ [B1]

Answer $x =$ ..... [1]

(b) An extra number y is added to the original list such that the median decreased by 2.
Find the value of y.

Answer .....$y = 6$ [B1]..... [1]

(c) A number is removed from the original list so that the median and range do not change.
Write down this number

Answer .....6 [B1]..... . [1]

25 The table shows the times (in minutes) taken by a class of students to travel from home to school daily.

Stem	Leaf					
0	5	8	9			
1	2	2	2	7	7	9
2	0	3	6	8		
3	0	2	5	7	8	
4	1	3	5	6		
5	4	6				

Key

1 | 2 represents 12 minutes

(a) Find the median time.

Answer .....27 minutes [B1].....[1]

(b) Find the percentage of students who took more than 30 minutes to travel to school daily.

$\frac{10}{24} \times 100 = 41.7\%$ or $41\frac{2}{3}\%$ [B1]

Answer ..... % [1]

(c) It was discovered that the times had been recorded incorrectly.
The correct times were all 2 minutes less than those recorded.
Explain how the median and range of the times have been affected by this error.
Answer .....

The median will decrease by 2 minutes (from 27 to 25 minutes) [B1]

The range will remain the same. [B1- using statistical definition of range]

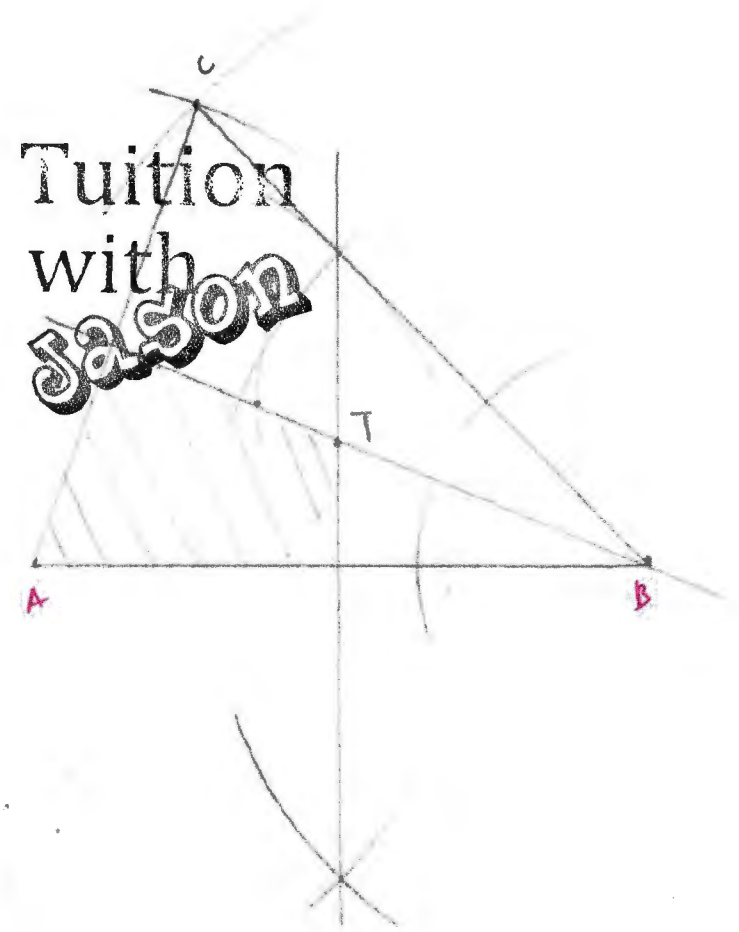
The new range of the times will be from 3min to 54min instead of the original 5min to 56min. [B1- using meaning of range as stated in Sec 2 textbook]

.....[2]

End of Paper

Setter: Mr Teo Lip Seng

AISS EOY 2022 P2 Qn 13



AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022

SECONDARY 3 EXPRESS

Name:	Class:	Register No.:
MARKING SCHEME		

MATHEMATICS
Paper 2
Candidates answer on the Question Paper.

4052/02
29 September 2022
2 hour 15 minutes

READ THESE INSTRUCTIONS FIRST

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For Examiner's Use
/ 90

This document consists of **21** printed pages.

Mathematical Formulae

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$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Alpha Data Company has the following promotion for tourists.

	7-day tourist SIM card	12-day tourist SIM card
First 100 GB of data	$\$p$	$\$(3p-4)$
Every additional GB of data	$\$(q+5)$	$\$q$

Tom bought a 7-day tourist SIM card and used 105 GB of data.
$62 was deducted from his credit card bill for his data usage after his vacation.
Claire bought a 12-day tourist SIM card and used 112 GB of data.
$92 was deducted from her credit card bill for her data usage after her vacation.

(a) Write down two simultaneous equations in terms of p and q, to represent this information.

$p+5(q+5)=62$
$p+5q=37$.....(1) B1
$3p-4+12q=92$
$3p+12q=96$.....(2) B1

[2]

(b) Solve the simultaneous equations.

(1)×3 $3p+15q=111$.....(3) M1
(3)−(2) $3q=15$
$q=5$ A1

Subst $q=5$ into (1)
$p+25=37$ A1
$p=12$

Answer $p=$ .....

$q=$ ..... [3]

2 **(a)** Write as a single fraction in its simplest form

(i) $\dfrac{2}{(x-5)^2}-\dfrac{3}{(5-x)}$,

$$\frac{2}{(x-5)^2}-\frac{3}{(5-x)}$$

$$=\frac{2}{(x-5)^2}+\frac{3(x-5)}{(x-5)^2}$$ M1

$$=\frac{3x-13}{(x-5)^2}$$ A1

Alternative Solution

$$\frac{2}{(x-5)^2}-\frac{3}{(5-x)}$$

$$=\frac{2}{(5-x)^2}-\frac{3(5-x)}{(5-x)^2}$$ M1

$$=\frac{3x-13}{(x-5)^2}$$ A1

Answer ..... [2]

(ii) $\dfrac{a^2-ab^2}{ab}\times\dfrac{ab^2}{ab-a^2}$.

$$\frac{a(a-b^2)}{ab}\times\frac{ab^2}{a(b-a)}$$ M1

$$=\frac{b(a-b^2)}{b-a}$$ A1

Answer ..... [2]

(b) Explain why $81x^2 + 4 = 0$ has no real solutions.

$81x^2 = -4$

$x^2 = -\frac{4}{81}$

Since $x^2 \geq 0$ for all real values of x, $81x^2 + 4 = 0$ has no real solutions.

B1

[1]

(c) Solve $\frac{3x-1}{2} - 8 = \frac{x+3}{5}$.

$\frac{3x-1}{2} - 8 = \frac{x+3}{5}$

$5(3x-1) - 80 = 2(x+3)$ M1

$15x - 5 - 80 = 2x + 6$ M1

$13x = 91$

$x = 7$ A1

Answer $x =$ ..... [3]

3 **(a)** Simplify $6^{2n+1} \times 8^n \times 3^{2n}$.

$2^{2n+1} \times 3^{2n+1} \times 2^{3n} \times 3^{2n}$ M1, M1

$= 2^{5n+1} \times 3^{4n+1}$ A1

Answer ..... [3]

(b) Solve $4^x\left(5^{2x}\right) = 10$.

$\left(2^{2x}\right)\left(5^{2x}\right) = 10$ M1

$10^{2x} = 10$ M1

$2x = 1$

$x = \frac{1}{2}$ A1

Answer $x =$ ..... [3]

(c) The reciprocal of $\sqrt[3]{32}$ is 4^n.
Find the exact value of n.

$$\frac{1}{32^{\frac{1}{3}}} = 4^n$$

$$\frac{1}{2^{\frac{5}{3}}} = 2^{2n}$$ M1

$$2^{\frac{-5}{3}} = 2^{2n}$$

$$2n = -\frac{5}{3}$$

$$n = -\frac{5}{6}$$ A1

Answer $n =$..... [2]

(d) List all the integers that satisfy the inequalities
$3 \le x - 3 \le 7$ and $\frac{x-1}{3} < \frac{2x+1}{5} < 4$.
Represent your answer on a number line.

$3 \le x - 3 \le 7$
$6 \le x \le 10$

$\frac{x-1}{3} < \frac{2x+1}{5}$ and $\frac{2x+1}{5} < 4$

$5x - 5 < 6x + 3$ $\quad$ $2x + 1 < 20$

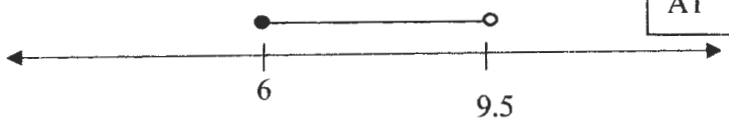
$x > -8$ M1 $\quad$ $x < 9.5$ M1

All 3 correct, M2
Minus 1 mark for each incorrect answer

$6 \le x < 9.5$ M1

Integers: 6, 7,8 and 9 A1

A1



Answer ..... [5]

4 Rui has a map drawn to the scale 1 : 200 000.

(a) The distance on the map between Gardens by the Bay and Night Safari is 14.1 cm
Calculate the actual distance, in kilometres, between Gardens by the Bay and Night Safari.

1 cm represents 2 km M1

14.1 cm represents 28.2 km A1

Answer .....km [2]

(b) Gardens by the Bay covers an area of 1.01 km^2.
Find the area, in square centimetres, covered by Gardens by the Bay on the map.

2 km represents 1 cm

4 km^2 represents 1 cm^2 M1

1.01 km^2 represents 0.2525 or $\frac{101}{400}$ cm^2 A1

Answer .....cm^2 [2]

(c) The area covered by Gardens by the Bay on Ting's map is 4 times that on Rui's map.
Find the scale of Ting's map, giving your answer in the form 1 : *n*.

Area on Ting's map = 4 (0.2525)

= 1.01 cm^2 M1

1.01 cm^2 represents 1.01 km^2

1 cm^2 represents 1 km^2

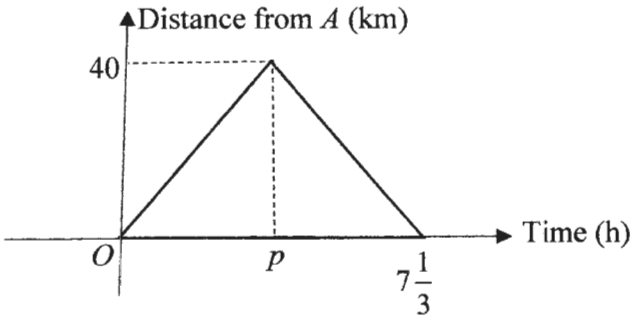
1 cm represents 1 km M1

1 cm represents 1 000 00 cm

1 : 100000 A1

Answer 1 : ..... [3]

5



A cyclist cycled a distance of 40 km from A to B at an average speed of x km/h for p hours where x and p are constants.
He then cycled from B to A by the same route but at an average speed of 2 km/h slower.
The total time taken by the cyclist for the whole journey was $7\frac{1}{3}$ hours.

(a) Express p in terms of x.

$p = \frac{40}{x}$ B1

Answer $y =$ ..... [1]

(b) Show that $\frac{22}{3} - p = \frac{40}{x-2}$ when he cycled from B to A.

$\frac{40}{\frac{22}{3} - p} = x - 2$ M1

$40 = (x-2)\left(\frac{22}{3} - p\right)$

$\frac{40}{x-2} = \frac{22}{3} - p$

$\frac{22}{3} - p = \frac{40}{x-2}$ (shown) A1

[2]

(c) Using the results parts **(a)** and **(b)**, show that $11x^2-142x+120=0$.

$\frac{22}{3}-\frac{40}{x}=\frac{40}{x-2}$ M1,subst p

$\frac{22}{3}=\frac{40}{x-2}+\frac{40}{x}$

$\frac{22}{3}=\frac{40x+40x-80}{x(x-2)}$ M1, reasonable attempt at combining fractions

$\frac{22}{3}=\frac{80x-80}{x(x-2)}$

$22x(x-2)=240x-240$

$22x^2-44x=240x-240$

$22x^2-284x+240=0$

$11x^2-142x+120=0$ (shown) A1

[3]

(d) Solve $11x^2-142x+120=0$.

$11x^2-142x+120=0$

$(x-12)(11x-10)=0$ M1

$x=12$ or $x=\frac{10}{11}$ or 0.909 A1, A1

Answer $x=$.....or $x=$.....[3]

(e) Explain why one of the solutions in part **(d)** must be rejected.

when $x=\frac{10}{11}$

$p=\left(\frac{40}{\frac{10}{11}}\right)$

$p=44$

Since p must be lesser than $7\frac{1}{3}$, $x=\frac{10}{11}$ must be rejected. B1

[1]

(f) Hence find the possible value of x and p.

$x = 12$ B1

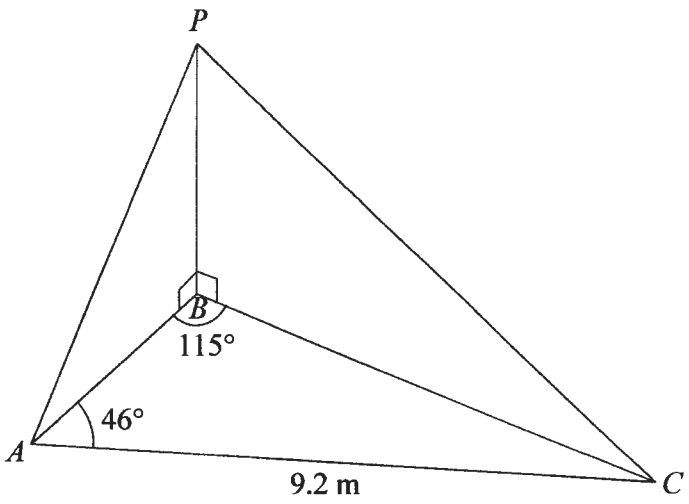
when $p = \frac{40}{12}$

$= 3\frac{1}{3}$ B1

Answer $x =$ .....

$p =$ ..... [2]

6



The diagram shows a field ABC on horizontal ground.
BP is a vertical pole vertically above B.
$AC = 9.2$ m, $BP = 5$ m, angle $BAC = 46°$ and angle $ABC = 115°$.

(a) Calculate AB.

$\angle ACB$
$= 180° - 115° - 46° (\angle \text{ sum of } \Delta)$ M1
$= 19°$

$\frac{AB}{\sin 19°} = \frac{9.2}{\sin 115°}$ M1

AB
$= \frac{9.2}{\sin 115°} \times \sin 19°$
$= 3.30487$
$= 3.30m$ A1

Answer ..... m [3]

(b) Tom walks along AC.
Find Tom's distance from A where he is equidistant from A and B.

Let the point where Tom is equidistant from A and B be T.
Triangle ABT is an isosceles triangle
$\angle ABT$
$= 180° - 46° - 46° (\angle \text{ sum of } \Delta)$ M1
$= 88°$

$$\frac{AT}{\sin 46°} = \frac{3.30487}{\sin 88°}$$
$$AT = \frac{3.30487}{\sin 88°} \times \sin 46° \quad \text{M1}$$
$$AT = 2.37877$$
$AT = 2.38\text{m}$ A1

Answer ..... m [3]

(c) The angle of depression of C from P is 34.4°.
Calculate CP.

$$\sin 34.4° = \frac{5}{CP}$$
$$CP = \frac{5}{\sin 34.4°} \quad \text{M1}$$
$$CP = 8.85007$$
$CP = 8.85\text{m}$ A1

Answer ..... m [2]

(d) Calculate angle ACP.

$$AP^2 = 5^2 + 3.304867357^2$$
$AP = 5.9935$ M1
$$\cos \angle ACP = \frac{9.2^2 + 8.8507^2 - 5.9935^2}{2(9.2)(8.8507)} \quad \text{M1}$$
$$\angle ACP = \cos^{-1}\left(\frac{9.2^2 + 8.8507^2 - 5.9935^2}{2(9.2)(8.8507)}\right)$$
$\angle ACP = 38.7239$
$\angle ACP = 38.7°$ A1

Answer Angle ACP = ... [3]

7 A group of researchers brought 180 kangaroos to an island and tracked the number of kangaroos over a few years.
Some corresponding values of x and y are given in the table below.

x (Number of years)	0	0.5	1	2	4	5	6	6.5	7	7.5
y (Number of kangaroos)	180	177	175	172	172	175	180	183	187	191

(a) On the grid opposite, plot the points given in the table and join them with a smooth curve. **(See graph paper for answers)**

B2 – Plotting all 10 points correctly
B1 – Smooth curve passing through all the points. [3]

(b) Use your graph to estimate the number of kangaroos at the third year, correct your answer to the nearest integer.

$k = 171$ B1

Answer ..... [1]

(c) By drawing a tangent, find the gradient of the curve at $x = 2$.

Draw a suitable tangent M1
Gradient
$= -2 \pm 0.2$
$(-2.2 \text{ to } -1.8)$ A1

$m = \dfrac{175 - 169.4}{0.3 - 3.4}$
$m = -1.81$ (3 s.f.)
Gradient based on graph drawn

Answer ..... [2]

(d) The researchers found out that the number of kangaroos can be modelled after a quadratic function of the form $y = (x + a)^2 + b$.
Find the value of a and b.

$y = (x - 3)^2 + 171$
$a = -3$ B1
$b = 171$ (based on answer from part b) B1

Answer $a =$ ....., $b =$ ..... [2]

(e) The researchers then brought 180 kangaroos of a different breed to another island and observed that the number of kangaroos increased at a constant rate of 1 per year.
Write down an equation connecting the number of kangaroos, y, and the number of years, x.

$y = x + 180$ B1

Answer ..... [1]

(f) On the same grid, draw the graph of the equation in part (e) for $4 \leq x \leq 8$. [1]

Draw line. B1
(See graph for answer)

(g) Explain the significance of the point of intersection of the two graphs.

Answer B1
At the 7th year, the number of kangaroos is the same (187 kangaroos) on both islands. [1]

---

8 Part of a pattern of numbers is shown in the table below.

	C_1	C_2	C_3	C_4	C_5	C_6
Row 1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13
Row 3	14	15	16	17	18	19
Row 4	20	21	22	23	24	25
Row 5	26	27	28	29	30	31
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Row n		a	b			
Row $n+1$		r	s			

(a) Express b, r and s in terms of a.

$b = a+1$
$r = a+6$
$s = a+7$

B3 (1 mark each)

Answer $b =$ .....
$r =$ .....
$s =$ ..... [3]

(b) Show that the difference between rb and as is always 6. [2]

Answer

rb
$= (a+1)(a+6)$
$= a^2 + 7a + 6$
as
$= a(a+7)$
$= a^2 + 7a$
$rb - as$
$= (a^2 + 7a + 6) - (a^2 + 7a)$
$= 6$ (shown)

M1

A1

(c) **(i)** Express a in terms of n.

$a = 6n - 3$ B1

Answer $a =$ ..... [1]

(ii) Express b in terms of n.

$b = 6n - 2$ B1

Answer $b =$ ..... [1]

(d) **(i)** Explain why ab will be in column number C_5, for any value of n.

ab
$= (6n-3)(6n-2)$
$= 36n^2 - 30n + 6$
$= 6(6n^2 - 5n + 1)$ M1

Since 6 is a factor of ab, ab will always be in C_5 for any n. A1

[2]

(ii) Which column would rs lie in?

rs
$= (6n-3+6)(6n-2+6)$
$= (6n+3)(6n+4)$
$= 36n^2 + 42n + 12$
$= 6(6n^2 + 7n + 2)$

C_5 B1

Answer ..... [1]

9

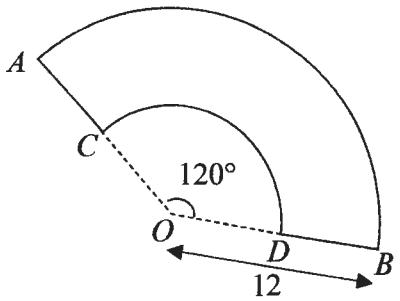


Diagram I

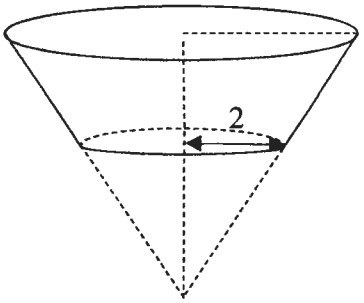


Diagram II

In diagram I, OAB is a sector of a circle, centre O, with radius 12 cm.
The region $CABD$ is cut from the sector and folded to form the frustum in Diagram II.
The small cone in diagram II has a radius of 2 cm.

(a) (i) Find the radius of the top of the frustum. [2]

Answer

Circumference of top of frustum = Arc length of sector OAB.

Circumference

$= \frac{120°}{360°} \times 2\pi(12)$

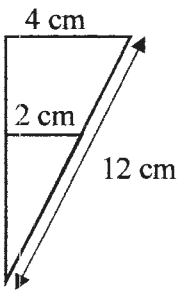
$= 8\pi$ cm M1

Radius

$= 8\pi \div 2\pi$

$= 4$ cm A1

(ii) Show that the volume of the frustum is $165.87\ \text{cm}^3$ when rounded to 5 significant figures. [4]



Slant height of cone = radius of the sector
The slant height of the bigger cone is 12 cm.

By Pythagoras' theorem,
Perpendicular height of bigger cone
$= \sqrt{12^2 - 4^2}$
$= \sqrt{128}\ \text{cm}$ M1

Let perpendicular height of smaller cone be x

$\frac{x}{\sqrt{128}} = \frac{2}{4}$

$x = \frac{1}{2} \times \sqrt{128}$

$x = \frac{\sqrt{128}}{2}$ M1

Volume of frustum
= volume of bigger cone – volume of smaller cone

$= \frac{1}{3}\pi(4^2)(\sqrt{128}) - \frac{1}{3}\pi(2^2)(\frac{\sqrt{128}}{2})$ M1

$= 165.86762$

$= 165.87\ \text{cm}^3 (3.s.f)$ A1

(b)

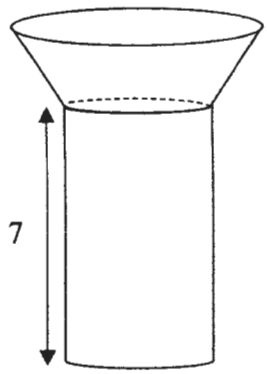
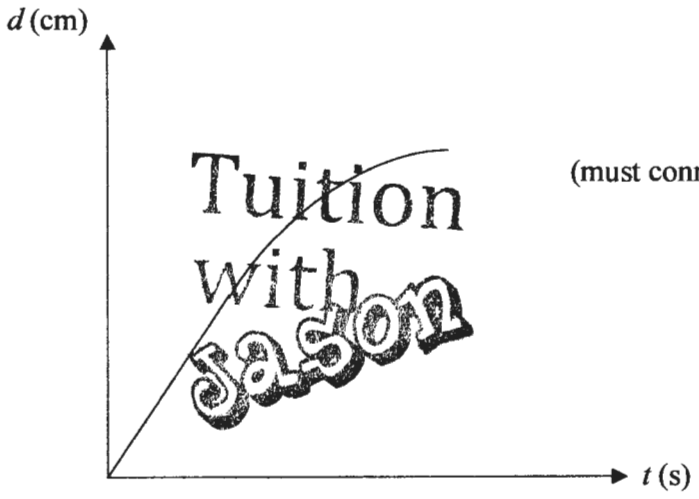


Diagram III

A cylinder is attached to the frustum to form a vase as shown in Diagram III.
Water is poured into the vase at a constant rate.

(i) On the axes, sketch the graph of the depth, d cm, of the water against time, t seconds.

Answer



B1 Shape
B1 Shape
(must connect smoothly)

[2]

(ii) The manufacturer wants to modify the vase such that the height of the cylinder is $\frac{6}{5}$ of its current height.
It takes 10 seconds to fill up the vase in diagram III.
The manufacturer claims that the time taken to fill the vase will approximately be 2.5 seconds more.
Assuming that the water is being filled up at the same rate, is the manufacturer correct?
Justify your answers with calculations. [5]

Volume of Vase
$= \pi(2^2)(7) + 165.8676297$
$= 253.8345943\text{cm}^3$ M1

Rate of filling up the vase
$= \frac{253.8345943}{10}$
$= 25.38345943\text{m}^3 / s$ M1

New volume
$= \pi(2^2)(\frac{6}{5} \times 7) + 165.87$
$= 271.4275132$ M1

Time taken
$= \frac{271.4275132}{25.38345943}$ M1
$= 10.69308594$
$= 10.7s$

Difference
$= 10.69308594 - 10$
$= 0.693s$

A1

No the manufacturer is incorrect as it only takes 0.693 seconds more to fill up the vase instead of 2.5 s.

END OF PAPER

(Answer)